

We seek to use lively minds, to work hard, to develop all our talents and to grow through sharing, to be the best version of ourselves



A LEVEL MATHS TRANSITION MATERIAL

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Refresher Course to Help with the Transition from GCSE to A-Level Mathematics

‘ Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. You go into the first room and it's dark, completely dark. You stumble around, bumping into the furniture. Gradually, you learn where each piece of furniture is. And finally, after six months or so, you find the light switch and turn it on. Suddenly, it's all illuminated and you can see exactly where you were. Then you enter the next dark room’... Sir Andrew Wiles

How to use this booklet

Thank you for choosing to study Mathematics in the sixth form at Caistor Grammar School. The Mathematics Department is committed to ensuring that you make good progress throughout your A-level course. In order that you make the best possible start to the course, we have prepared this booklet. The material in this refresher course has been designed to enable you to prepare for the demands of A-Level maths.

When your course starts in September you will find that your ability to get the most from lessons, and to understand new material, depends crucially upon both having a good facility with algebraic manipulation and undertaking plenty of independent study. We think that this is so important that we are providing you with this workbook to complete before you start the course in September. It is vitally important that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics before you commence the course in September.

A-Level maths is a demanding course and good skills in the areas covered within this booklet will be paramount to your success. You will most-likely have met all the topics before at GCSE. You should attempt, at least, most questions in each exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly. The answers are given at the back of the booklet, and you should mark your work. If you get questions wrong, you should revise the material and try again until you are getting most questions correct. You may find some questions towards the end of each section quite tricky – this is ok, but still try them! In September, you will be asked about your progress with the booklet. You will also sit a test at the start of September to check how well you understand these topics, so it is important that you have worked through the booklet before then. A mock test is provided at the back of this booklet, with answers. We hope that you will use this introduction to give you a good start to your A-Level work and that it will help you enjoy the course more.

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We would recommend Dr Frost Maths as a means of finding online notes and questions. Once you are part of the sixth form you will receive an account linked to your school email address. However, many resources can be accessed without an account.

Getting Help

You may also find the following books useful in helping you to prepare for your year 12 studies.
Head Start to AS Maths Preparation for AS/A Level Mathematics Published by CGP Workbooks
www.cgpbooks.co.uk

Preparation for A Level Maths Published by Alpha Workbooks
www.alphaworkbooks.co.uk

The course

We will be studying the AQA A Level Mathematics course 7357.
This involves 3 exams (2 hours each), taken at the end of year 13

- Paper 1: Pure
- Paper 2: Pure and Mechanics
- Paper 3: Pure and Statistics

There will exams at the end of year 12 as well as regular topic tests, these do not count towards your final grade but will be used as a basis for UCAS predictions at the end of year 12.

Calculators

We recommend the Casio FX991CW calculator for the course – your GCSE calculator is not likely to have enough features for the A Level course, so you will need to upgrade at some point. These cost from £20-30. We hope to make these available through the school library in September. Graphics calculators are also allowed in the exams, but these are more expensive, but can prove useful, particularly for Further Mathematics.

1. Arithmetic of Fractions

 **MyMaths.co.uk** Number > Fractions > Adding Subtracting, Multiplying, Dividing

1. Express each of the following as a fraction in its simplest form. For example $\frac{3}{21}$ can be written as $\frac{1}{7}$. Remember, no calculators!

a) $\frac{20}{45}$ b) $\frac{16}{36}$ c) $-\frac{42}{21}$ d) $\frac{18}{16}$ e) $\frac{30}{30}$ f) $\frac{17}{21}$ g) $-\frac{49}{35}$ h) $\frac{90}{30}$

2. Calculate

a) $\frac{1}{2} + \frac{1}{3}$ b) $\frac{1}{2} - \frac{1}{3}$ c) $\frac{2}{3} + \frac{3}{4}$ d) $\frac{5}{6} - \frac{2}{3}$ e) $\frac{8}{9} + \frac{1}{5} + \frac{1}{6}$ f) $\frac{4}{5} + \frac{3}{7} - \frac{9}{10}$

3. Evaluate the following, expressing each answer in its simplest form.

a) $\frac{4}{5} \times \frac{3}{16}$ b) $2 \times 3 \times \frac{1}{4}$ c) $\frac{3}{4} \times \frac{3}{4}$ d) $\frac{4}{9} \times 6$ e) $\frac{15}{16} \times \frac{4}{5}$ f) $\frac{9}{5} \times \frac{1}{3} \times \frac{15}{27}$

4. Evaluate

a) $3 \div \frac{1}{2}$ b) $\frac{1}{2} \div \frac{1}{4}$ c) $\frac{6}{7} \div \frac{16}{21}$ d) $\frac{3}{4}$ e) $5 \div \frac{10}{9}$ f) $\frac{3}{4} \div \frac{4}{3}$

5. Express the following as mixed fractions. A mixed fraction has a whole number part and a fractional part. For example, $\frac{13}{5}$ can be written as the mixed fraction $2\frac{3}{5}$.

a) $\frac{5}{2}$ b) $\frac{7}{3}$ c) $-\frac{11}{4}$ d) $\frac{6}{5}$ e) $\frac{12}{5}$ f) $\frac{18}{7}$ g) $\frac{16}{3}$ h) $\frac{83}{9}$

6. Express the following as improper fractions. An improper fraction is 'top-heavy'. Its numerator is greater than its denominator. For example, the mixed fraction $13\frac{4}{5}$ can be written as the improper fraction $\frac{69}{5}$.

a) $2\frac{1}{4}$ b) $3\frac{1}{2}$ c) $5\frac{2}{3}$ d) $-3\frac{2}{5}$ e) $11\frac{4}{6}$ f) $8\frac{2}{9}$ g) $16\frac{3}{4}$ h) $89\frac{2}{7}$

2. Rules and Manipulation of Indices

 **MyMaths.co.uk** Number > Powers and Roots > Indices 1, 2 and 3

1. Simplify the following algebraic expressions.

- a) $x^3 \times x^4$ b) $y^2 \times y^3 \times y^5$ c) $z^3 \times z^2 \times z$ d) $t^2 \times t^{10} \times t$
e) $a \times a \times a^2$ f) $t^3 t^4$ g) $b^6 b^3 b$ h) $z^7 z^7$

2. Simplify

- a) $\frac{x^6}{x^2}$ b) $\frac{y^{14}}{y^{10}}$ c) $\frac{t^{16}}{t^{12}}$ d) $\frac{z^{10}}{z^9}$ e) $\frac{v^7}{v^0}$ f) x^7/x^4

3. Simplify the following:

- a) $\frac{10^7}{10^6}$ b) $\frac{10^{19}}{10^{16}}$ c) $\frac{x^7}{x^{14}}$ d) $\frac{x^7}{y^4}$
e) $\frac{(ab)^4}{a^2 b^2}$ f) $\frac{9^9 10^{10}}{10^9}$ g) $\frac{x^9 y^8}{y^7 x^6}$ h) $\frac{(abc)^3}{(abc)^2}$

4. Write the following expressions using only *positive* indices. For example $\frac{x^{-4}}{x^{-2}}$ can be written as $\frac{1}{x^2}$.

a) $x^{-2}x^{-1}$ b) $\frac{3x}{x^{-4}}$ c) $\frac{t^{-2}}{t^{-3}}$
d) $(2a^2b^3)(6a^{-3}b^{-5})$ e) $\frac{x^{-3}}{5^{-2}}$ f) $\frac{(27)^{-1}x^{-1}}{y^{-2}}$

5. Without using a calculator, evaluate

a) $\frac{3}{4^{-2}}$ b) 4×3^{-2} c) $3^{-1}9^2(27)^{-1}$
d) $(0.25)^{-1}$ e) $(0.2)^{-2}$ f) $(0.1)^{-3}$

6. Simplify

a) $t^{-6}t^3$ b) $(-y^{-2})(-y^{-1})$ c) $\frac{3y^{-2}}{6y^{-3}}$ d) $(-2t^{-1})(-3t^{-2})(-4t^{-3})$
e) $\frac{3t^{-2}}{6t^3}$ f) $\frac{(2t^{-1})^3}{6t^2}$ g) $\frac{(-2t)^3}{(-4t)^2}$

7. Write the following expressions using a *single* index. For example $(5^3)^{-4}$ can be written as 5^{-12} .

- a) $(5^3)^5$ b) $(3^3)^3$ c) $(17^2)^4$ d) $(y^3)^6$ e) $(\frac{y^{-1}}{y^{-2}})^3$
f) $(\frac{t^{-2}}{t^4})^3$ g) $(k^{-2})^{-6}$ h) $((-1)^4)^3$ i) $((-1)^{-4})^{-3}$

8. Without the use of a calculator, evaluate

- a) $(4^{-1})^2$ b) $(2^2)^{-1}$ c) $(3^2)^2$ d) $(6^{-2})^{-1}$
e) $(\frac{2}{5^2})^{-1}$ f) $(-2)^{-1}$ g) $(-\frac{2}{3})^{-2}$

9. Write the following expressions without using brackets.

a) $(4^2 5^3)^3$ b) $\left(\frac{3ab}{c^3}\right)^2$ c) $\left(\frac{4^{-2}a^{-3}}{b^{-1}}\right)^2$ d) $(2a^2b)^3$
e) $(3xy^2z^3)^2$ f) $\left(\frac{6}{ab^2}\right)^2$ g) $\left(-\frac{3}{x^2}\right)^2$ h) $\left(\frac{2z^2}{3t}\right)^3$
i) $(-2x)^2$ j) $(-2x^2)^{-2}$ k) $\left(-\frac{2}{x^2}\right)^{-3}$

10. Write the following expressions without using brackets.

a) $(6^{1/2})^3$ b) $(5^{1/3})^6$ c) $(10^{0.6})^4$ d) $(x^2)^{1/3}$
e) $(2x^2)^{1/3}$ f) $(a \times a^2)^{1/2}$ g) $(ab^2)^{1/2}$

11. Write the following expressions without using brackets.

a) $(4^3)^{-1/2}$ b) $(3^{-1/2})^{-1/2}$ c) $(7^{2/3})^4$ d) $(19^{3/2})^{1/3}$

e) $(a^2b^{-3})^{-\frac{3}{2}}$ f) $\left(\frac{k^{-1.5}}{\sqrt{k}}\right)^{-2}$

12. Write the following expressions without using brackets.

a) $(5b)^{1/6}$ b) $(3\sqrt{x})^3$ c) $3(\sqrt{x})^3$ d) $(\sqrt{3x})^3$

13. Simplify

a) $x^{1/2}x^{1/3}$ b) $\frac{x^{1/2}}{x^{1/3}}$ c) $(x^{1/2})^{1/3}$ d) $(8x^3)^{1/3}$

e) $\sqrt{25y^2}$ f) $\left(\frac{27}{t^3}\right)^{1/3}$ g) $(16y^4)^{1/4}$ h) $(x^{1/4}x^{1/2})^4$

i) $\sqrt{a^2a^6}$ j) $\sqrt{\frac{a^{-4}}{a^{-1}}}$

3. Expanding Brackets and Factorising

 **MyMaths.co.uk** Algebra > Algebraic Manipulation > Single Brackets, Brackets, Factorising Linear, Factorising Quadratics 1 and 2

1. Write the following expressions without using brackets:

- a) $2(mn)$ b) $2(m + n)$ c) $a(mn)$ d) $a(m + n)$ e) $a(m - n)$
f) $(am)n$ g) $(a + m)n$ h) $(a - m)n$ i) $5(pq)$ j) $5(p + q)$
k) $5(p - q)$ l) $7(xy)$ m) $7(x + y)$ n) $7(x - y)$ o) $8(2p + q)$
p) $8(2pq)$ q) $8(2p - q)$ r) $5(p - 3q)$ s) $5(p + 3q)$ t) $5(3pq)$

2. Write the following expressions without using brackets and simplify where possible:

- a) $(2 + a)(3 + b)$ b) $(x + 1)(x + 2)$ c) $(x + 3)(x + 3)$ d) $(x + 5)(x - 3)$

3. Write the following expressions without using brackets:

- a) $(7 + x)(2 + x)$ b) $(9 + x)(2 + x)$ c) $(x + 9)(x - 2)$ d) $(x + 11)(x - 7)$
e) $(x + 2)x$ f) $(3x + 1)x$ g) $(3x + 1)(x + 1)$ h) $(3x + 1)(2x + 1)$
i) $(3x + 5)(2x + 7)$ j) $(3x + 5)(2x - 1)$ k) $(5 - 3x)(x + 1)$ l) $(2 - x)(1 - x)$

4. Rewrite the following expressions without using brackets:

- a) $(s + 1)(s + 5)(s - 3)$ b) $(x + y)^3$

5. Factorise

- a) $5x + 15y$ b) $3x - 9y$ c) $2x + 12y$ d) $4x + 32z + 16y$ e) $\frac{1}{2}x + \frac{1}{4}y$

6. Factorise

a) $\frac{1}{3}x + \frac{1}{6}xy$ b) $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ c) $a^2 - a + \frac{1}{4}$ d) $\frac{1}{x^2} - \frac{2}{x} + 1$

7. Factorise

a) $x^2 + 8x + 7$ b) $x^2 + 6x - 7$ c) $x^2 + 7x + 10$ d) $x^2 - 6x + 9$ e) $x^2 + 5x + 6$.

8. Factorise

a) $2x^2 + 3x + 1$ b) $2x^2 + 4x + 2$ c) $3x^2 - 3x - 6$ d) $5x^2 - 4x - 1$
e) $16x^2 - 1$ f) $-x^2 + 1$ g) $-2x^2 + x + 3$

9. Factorise

- a) $x^2 + 9x + 14$ b) $x^2 + 11x + 18$ c) $x^2 + 7x - 18$ d) $x^2 + 4x - 77$
e) $x^2 + 2x$ f) $3x^2 + x$, g) $3x^2 + 4x + 1$ h) $6x^2 + 5x + 1$
i) $6x^2 + 31x + 35$ j) $6x^2 + 7x - 5$ k) $-3x^2 + 2x + 5$ l) $x^2 - 3x + 2$

10. Rewrite the following expressions without using brackets, simplifying where possible:

- a) $15 - (7 - x)$ b) $15 - 7(1 - x)$
c) $15 - 7(x - 1)$ d) $(2x - y) - x(1 + y)$
e) $x(a + b) - x(a + 3b)$ f) $2(5a + 3b) + 3(a - 2b)$
g) $-(4a + 5b - 3c) - 2(2a + 3b - 4c)$ h) $2x(x - 5) - x(x - 2) - 3x(x - 5)$

11. Rewrite each of the following expressions without using brackets and simplify where possible

a) $2x - (3y + 8x)$, b) $2x + 5(x - y - z)$, c) $-(5x - 3y)$, d) $5(2x - y) - 3(x + 2y)$

4. Algebraic Fractions

 **MyMaths.co.uk** Algebra > Algebraic Manipulation > Cancelling. Adding and Multiplying Algebraic Fractions

1. Express each of the following as a single fraction.

a) $2 \times \frac{x+y}{3}$ b) $\frac{1}{3} \times 2(x+y)$ c) $\frac{2}{3} \times (x+y)$

2. Simplify

a) $3 \times \frac{x+4}{7}$ b) $\frac{1}{7} \times 3(x+4)$ c) $\frac{3}{7} \times (x+4)$ d) $\frac{x}{y} \times \frac{x+1}{y+1}$
e) $\frac{1}{y} \times \frac{x^2+x}{y+1}$ f) $\frac{\pi d^2}{4} \times \frac{Q}{\pi d^2}$ g) $\frac{Q}{\pi d^2/4}$ h) $\frac{1}{x/y}$

3. Simplify a) $\frac{6/7}{s+3}$ b) $\frac{3/4}{x-1}$ c) $\frac{x-1}{3/4}$

4. Simplify $\frac{3}{x+2} \div \frac{x}{2x+4}$

5. Simplify $\frac{5}{2x+1} \div \frac{x}{3x-1}$

6. Simplify

a) $\frac{x}{4} + \frac{x}{7}$ b) $\frac{2x}{5} + \frac{x}{9}$ c) $\frac{2x}{3} - \frac{3x}{4}$ d) $\frac{x}{x+1} - \frac{2}{x+2}$
 e) $\frac{x+1}{x} + \frac{3}{x+2}$ f) $\frac{2x+1}{3} - \frac{x}{2}$ g) $\frac{x+3}{2x+1} - \frac{x}{3}$ h) $\frac{x}{4} - \frac{x}{5}$

7. Simplify

a) $\frac{1}{x+2} + \frac{2}{x+3}$ b) $\frac{2}{x+3} + \frac{5}{x+1}$ c) $\frac{2}{2x+1} - \frac{3}{3x+2}$
 d) $\frac{x+1}{x+3} + \frac{x+4}{x+2}$ e) $\frac{x-1}{x-3} + \frac{x-1}{(x-3)^2}$

5. Surds

MyMaths.co.uk Number > Powers and Roots > Surds 1 and Surds 2

Roots, for example $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{6}$ are also known as surds. A common cause of error is misuse of expressions involving surds. You should be aware that $\sqrt{ab} = \sqrt{a}\sqrt{b}$ but $\sqrt{a+b}$ is NOT equal to $\sqrt{a} + \sqrt{b}$.

1. It is often possible to write surds in equivalent forms. For example $\sqrt{48}$ can be written $\sqrt{3 \times 16} = \sqrt{3} \times \sqrt{16} = 4\sqrt{3}$.

Write the following in their simplest forms:

- a) $\sqrt{180}$ b) $\sqrt{63}$

2. By multiplying numerator and denominator by $\sqrt{2} + 1$ show that

$$\frac{1}{\sqrt{2} - 1} \quad \text{is equivalent to} \quad \sqrt{2} + 1$$

3. Simplify, if possible, a) $\sqrt{x^2 y^2}$ b) $\sqrt{x^2 + y^2}$.

4. Study the following expressions and simplify where possible.

a) $\sqrt{(x+y)^4}$ b) $(\sqrt[3]{x+y})^6$ c) $\sqrt{x^4+y^4}$

5. By considering the expression $(\sqrt{x} + \sqrt{y})^2$ show that

$$\sqrt{x} + \sqrt{y} = \sqrt{x+y+2\sqrt{xy}}$$

Find a corresponding expression for $\sqrt{x} - \sqrt{y}$.

6. Write each of the following as an expression under a single square root sign. (For parts c) and d) see Question 5 above.)

a) $2\sqrt{p}$ b) $\sqrt{p}\sqrt{q^3}$ c) $\sqrt{p} + \sqrt{2q}$ d) $\sqrt{3} - \sqrt{2}$

7. Use indices (powers) to write the following expressions without the root sign.

a) $\sqrt[4]{a^2}$ b) $(\sqrt{3} \times \sqrt{5})^3$

6. Linear Equations

 **MyMaths.co.uk** Algebra > Equations – Linear > Solving Equations, Equations with Fractions

In questions 1 – 35 solve each equation:

1. $3y - 8 = \frac{1}{2}y$ 2. $7t - 5 = 4t + 7$ 3. $3x + 4 = 4x + 3$ 4. $4 - 3x = 4x + 3$

5. $3x + 7 = 7x + 2$ 6. $3(x + 7) = 7(x + 2)$ 7. $2x - 1 = x - 3$ 8. $2(x + 4) = 8$

9. $-2(x - 3) = 6$ 10. $-2(x - 3) = -6$
11. $-3(3x - 1) = 2$ 12. $2 - (2t + 1) = 4(t + 2)$

13. $5(m - 3) = 8$ 14. $5m - 3 = 5(m - 3) + 2m$
15. $2(y + 1) = -8$ 16. $17(x - 2) + 3(x - 1) = x$

$$17. \frac{1}{3}(x+3) = -9 \quad 18. \frac{3}{m} = 4 \quad 19. \frac{5}{m} = \frac{2}{m+1} \quad 20. -3x + 3 = 18$$

$$21. 3x + 10 = 31 \quad 22. x + 4 = \sqrt{8} \quad 23. x - 4 = \sqrt{23}$$

$$24. \frac{x-5}{2} - \frac{2x-1}{3} = 6 \quad 25. \frac{x}{4} + \frac{3x}{2} - \frac{x}{6} = 1$$

$$26. \frac{x}{2} + \frac{4x}{3} = 2x - 7 \quad 27. \frac{5}{3m+2} = \frac{2}{m+1}$$

$$28. \frac{2}{3x-2} = \frac{5}{x-1} \quad 29. \frac{x-3}{x+1} = 4 \quad 30. \frac{x+1}{x-3} = 4 \quad 31. \frac{y-3}{y+3} = \frac{2}{3}$$

$$32. \frac{4x+5}{6} - \frac{2x-1}{3} = x \quad 33. \frac{3}{2s-1} + \frac{1}{s+1} = 0$$

$$34. \frac{1}{5x} + \frac{1}{4x} = 10. \quad 35. \frac{3}{s-1} = \frac{2}{s-5}.$$

7. Changing the Subject of a Formulae

 **MyMaths.co.uk** Algebra > Expressions and Formulae > Rearranging 1 and Rearranging 2

1. Make t the subject of the formula $p = \frac{c}{\sqrt{t}}$.

2. Make N the subject of the formula $L = \frac{\mu N^2 A}{\ell}$.

3. In each case make the specified variable the subject of the formula:

a) $h = c + d + 2e$, e b) $S = 2\pi r^2 + 2\pi rh$, h

c) $Q = \sqrt{\frac{c+d}{c-d}}$, c d) $\frac{x+y}{3} = \frac{x-y}{7} + 2$, x

4. Make n the subject of the formula $J = \frac{nE}{nL + m}$.

8. Solving Quadratic Equations – Factorising

 **MyMaths.co.uk** Algebra > Equations – Quadratic > Quadratic Equations

Solve the following equations by factorisation:

1. $x^2 - 3x + 2 = 0$ 2. $x^2 - x - 2 = 0$ 3. $x^2 + x - 2 = 0$ 4. $x^2 + 3x + 2 = 0$

5. $x^2 + 8x + 7 = 0$ 6. $x^2 - 7x + 12 = 0$ 7. $x^2 - x - 20 = 0$ 8. $x^2 - 1 = 0$

9. $x^2 - 2x + 1 = 0$ 10. $x^2 + 2x + 1 = 0$ 11. $x^2 + 11x = 0$ 12. $2x^2 + 2x = 0$

13. $x^2 - 3x = 0$ 14. $x^2 + 9x = 0$ 15. $2x^2 - 5x + 2 = 0$ 16. $6x^2 - x - 1 = 0$

17. $-5x^2 + 6x - 1 = 0$ 18. $-x^2 + 4x - 3 = 0$

9. Solving Quadratic Equations – Completing the Square and Using the Quadratic Formula

 **MyMaths.co.uk** Algebra > Equations – Quadratic > Completing the Square and Quadratic Formula

Solve each of the following quadratic equations twice: once by using the formula, then again by completing the square. Obtain your answers in surd, not decimal, form.

1. $x^2 + 8x + 1 = 0$
2. $x^2 + 7x - 2 = 0$
3. $x^2 + 6x - 2 = 0$
4. $4x^2 + 3x - 2 = 0$
5. $2x^2 + 3x - 1 = 0$
6. $x^2 + x - 1 = 0$
7. $-x^2 + 3x + 1 = 0$
8. $-2x^2 - 3x + 1 = 0$
9. $2x^2 + 5x - 3 = 0$
10. $-2s^2 - s + 3 = 0$
11. $9x^2 + 16x + 1 = 0$
12. $x^2 + 16x + 9 = 0$

13. Show that the roots of $x^2 - 2x + \alpha = 0$ are $x = 1 + \sqrt{1 - \alpha}$ and $x = 1 - \sqrt{1 - \alpha}$.

14. Show that the roots of $x^2 - 2\alpha x + \beta = 0$ are

$$x = \alpha + \sqrt{\alpha^2 - \beta} \quad \text{and} \quad x = \alpha - \sqrt{\alpha^2 - \beta}$$

10. Solving Simultaneous Linear Equations

 **MyMaths.co.uk** Algebra > Equations – Simultaneous > Simultaneous 1, 2, 3 and Negative

Solve each of the pairs of simultaneous equations. Where the answer is not a whole number, give your answer as an improper fraction.

1. $4x + 3y = 6$
 $5x - 3y = 21$

4. $2a + b = 3$
 $4a - 5b = 20$

7. $3a - 2b = 14$
 $4a + 3b = 13$

2. $3x + 5y = 13$
 $2x + 3y = 8$

5. $5x + 3y = 4$
 $3x + 4y = 9$

8. $5x + 4y = 5$
 $2x + 7y = 29$

3. $x + 4y = 5$
 $4x - 2y = 11$

6. $6x - 2y = 13$
 $2x + 3y = -3$

9. $6x - 4y = 39$
 $2x + y = 6$

Answers

Section 1. Arithmetic of fractions

1. a) $\frac{4}{9}$, b) $\frac{4}{9}$, c) -2 , d) $\frac{9}{8}$, e) 1 , f) $\frac{17}{21}$, g) $-\frac{7}{5}$, h) 3
2. a) $\frac{5}{6}$, b) $\frac{1}{6}$, c) $\frac{17}{12}$, d) $\frac{1}{6}$, e) $\frac{113}{90}$, f) $\frac{23}{70}$
3. a) $\frac{3}{20}$, b) $\frac{3}{2}$, c) $\frac{9}{16}$, d) $\frac{8}{3}$, e) $\frac{3}{4}$, f) $\frac{1}{3}$.
4. a) 6 , b) 2 , c) $\frac{9}{8}$, d) $\frac{3}{16}$, e) $\frac{9}{2}$, f) $\frac{9}{16}$
5. a) $2\frac{1}{2}$, b) $2\frac{1}{3}$, c) $-2\frac{3}{4}$, d) $1\frac{1}{5}$, e) $2\frac{2}{5}$, f) $2\frac{4}{7}$, g) $5\frac{1}{3}$, h) $9\frac{2}{9}$
6. a) $\frac{9}{4}$, b) $\frac{7}{2}$, c) $\frac{17}{3}$, d) $-\frac{17}{5}$, e) $\frac{35}{3}$, f) $\frac{74}{9}$, g) $\frac{67}{4}$, h) $\frac{625}{7}$

Section 2. Manipulation of expressions involving indices

1. a) x^7 , b) y^{10} , c) z^6 , d) t^{13} , e) a^4 , f) t^7 , g) b^{10} , h) z^{14} .
2. a) x^4 , b) y^4 , c) t^4 , d) z , e) v^7 , f) x^3
3. a) 10 , b) 10^3 , c) x^{-7} , d) $\frac{x^7}{y^4}$, e) a^2b^2 , f) $9^9 \cdot 10$, g) x^3y , h) abc
4. a) $\frac{1}{x^3}$, b) $3x^5$, c) t , d) $\frac{12}{ab^2}$, e) $\frac{5^2}{x^3}$, f) $\frac{y^2}{27x}$
5. a) 48 , b) $\frac{4}{9}$, c) 1 , d) 4 , e) 25 , f) 1000
6. a) t^{-3} , b) y^{-3} , c) $\frac{1}{2}y$, d) $-24t^{-6}$, e) $\frac{1}{2t^5}$, f) $\frac{4t^{-5}}{3}$, g) $-\frac{t}{2}$.
7. a) 5^{15} , b) 3^9 , c) 17^8 , d) y^{18} , e) y^3 , f) t^{-18} , g) k^{12} , h) $(-1)^{12} = 1$, i) $(-1)^{12} = 1$.
8. a) $\frac{1}{16}$, b) $\frac{1}{4}$, c) 81 , d) 36 , e) $\frac{25}{2}$, f) $-\frac{1}{2}$, g) $\frac{9}{4}$
9. a) 4^65^9 , b) $\frac{9a^2b^2}{c^6}$, c) $\frac{4^{-4}a^{-6}}{b^{-2}} = \frac{b^2}{4^4a^6}$, d) $8a^6b^3$, e) $9x^2y^4z^6$, f) $\frac{36}{a^2b^4}$,
g) $\frac{9}{x^4}$, h) $\frac{8z^6}{27t^3}$, (i) $4x^2$, j) $\frac{1}{4x^4}$, k) $-\frac{x^6}{8}$.
10. a) $6^{3/2}$, b) 5^2 , c) $10^{2.4}$, d) $x^{2/3}$, e) $2^{1/3}x^{2/3}$, f) $a^{3/2}$, g) $a^{1/2}b$.
11. a) $4^{-3/2}$, b) $3^{1/4}$, c) $7^{8/3}$, d) $19^{1/2}$, e) $a^{-3}b^{9/2}$, f) k^4 .
12. a) $5^{1/6}b^{1/6}$, b) $27x^{3/2}$, c) $3x^{3/2}$, d) $3^{3/2}x^{3/2}$
13. a) $x^{5/6}$, b) $x^{1/6}$, c) $x^{1/6}$, d) $2x$, e) $5y$, f) $\frac{3}{t}$, g) $2y$, h) x^3 , i) a^4 , j) $a^{-3/2}$

Section 3. Removing brackets and factorisation

1. a) $2mn$, b) $2m + 2n$, c) amn , d) $am + an$, e) $am - an$, f) amn , g) $an + mn$,
h) $an - mn$, i) $5pq$, j) $5p + 5q$, k) $5p - 5q$, l) $7xy$, m) $7x + 7y$, n) $7x - 7y$,
o) $16p + 8q$, p) $16pq$, q) $16p - 8q$, r) $5p - 15q$, s) $5p + 15q$, t) $15pq$
2. a) $6 + 3a + 2b + ab$, b) $x^2 + 3x + 2$, c) $x^2 + 6x + 9$, d) $x^2 + 2x - 15$
3. a) $14 + 9x + x^2$, b) $18 + 11x + x^2$, c) $x^2 + 7x - 18$,
d) $x^2 + 4x - 77$, e) $x^2 + 2x$, f) $3x^2 + x$, g) $3x^2 + 4x + 1$
h) $6x^2 + 5x + 1$, i) $6x^2 + 31x + 35$, j) $6x^2 + 7x - 5$
k) $-3x^2 + 2x + 5$, l) $x^2 - 3x + 2$
4. a) $s^3 + 3s^2 - 13s - 15$, b) $x^3 + 3x^2y + 3xy^2 + y^3$

5. a) $5(x+3y)$, b) $3(x-3y)$, c) $2(x+6y)$, d) $4(x+8z+4y)$, e) $\frac{1}{2}(x+\frac{1}{2}y)$
6. a) $\frac{x}{3}\left(1+\frac{y}{2}\right)$, b) $\frac{\pi r^2}{3}(2r+h)$, c) $\left(a-\frac{1}{2}\right)^2$, d) $\left(\frac{1}{x}-1\right)^2$.
7. a) $(x+7)(x+1)$, b) $(x+7)(x-1)$, c) $(x+2)(x+5)$, d) $(x-3)(x-3) = (x-3)^2$,
e) $(x+3)(x+2)$
8. a) $(2x+1)(x+1)$, b) $2(x+1)^2$, c) $3(x+1)(x-2)$, d) $(5x+1)(x-1)$,
e) $(4x+1)(4x-1)$, f) $(x+1)(1-x)$, g) $(x+1)(3-2x)$
9. a) $(7+x)(2+x)$, b) $(9+x)(2+x)$, c) $(x+9)(x-2)$, d) $(x+11)(x-7)$,
e) $(x+2)x$, f) $(3x+1)x$, g) $(3x+1)(x+1)$, h) $(3x+1)(2x+1)$ i) $(3x+5)(2x+7)$,
j) $(3x+5)(2x-1)$, k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$
10. a) $8+x$, b) $8+7x$, c) $22-7x$, d) $x-y-xy$, e) $-2bx$, f) $13a$, g) $-8a-11b+11c$,
h) $7x-2x^2$.
11. a) $-3y-6x$, b) $7x-5y-5z$, c) $-5x+3y$, d) $7x-11y$.

Section 4. Arithmetic of Algebraic Fractions

1. a) $\frac{2(x+y)}{3}$, b) $\frac{2(x+y)}{3}$, c) $\frac{2(x+y)}{3}$
2. a) $\frac{3(x+4)}{7}$, b) $\frac{3(x+4)}{7}$, c) $\frac{3(x+4)}{7}$, d) $\frac{x(x+1)}{y(y+1)}$, e) $\frac{x(x+1)}{y(y+1)}$, f) $Q/4$, g) $\frac{4Q}{\pi d^2}$, h) $\frac{y}{x}$.
3. a) $\frac{6}{7(s+3)}$, b) $\frac{3}{4(x-1)}$, c) $\frac{4(x-1)}{3}$.
4. $\frac{6}{x}$. 5. $\frac{5(3x-1)}{x(2x+1)}$
6. a) $\frac{11x}{28}$, b) $\frac{23x}{45}$, c) $-\frac{x}{12}$, d) $\frac{x^2-2}{(x+1)(x+2)}$, e) $\frac{x^2+6x+2}{x(x+2)}$, f) $\frac{x+2}{6}$, g) $\frac{9+2x-2x^2}{3(2x+1)}$, h) $\frac{x}{20}$.
7. a) $\frac{3x+7}{(x+2)(x+3)}$, b) $\frac{7x+17}{(x+3)(x+1)}$, c) $\frac{1}{(2x+1)(3x+2)}$, d) $\frac{2x^2+10x+14}{(x+3)(x+2)}$, e) $\frac{x^2-3x+2}{(x-3)^2}$

Section 5. Surds

1. a) $\sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}$. b) $\sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$.
3. a) xy . b) Note that $\sqrt{x^2+y^2}$ is NOT equal to $x+y$.
4. a) $(x+y)^2$, b) $(x+y)^2$, c) $\sqrt{x^4+y^4}$ is NOT equal to x^2+y^2 .
5. $\sqrt{x}-\sqrt{y} = \sqrt{x+y-2\sqrt{xy}}$.
6. a) $\sqrt{4p}$, b) $\sqrt{pq^3}$, c) $\sqrt{p+2q+2\sqrt{2pq}}$, d) $\sqrt{5-2\sqrt{6}}$.
7. a) $a^{1/2}$, b) $15^{3/2}$

Section 6. Solving linear equations

1. $16/5$, 2. 4 , 3. 1 , 4. $1/7$,
5. $5/4$, 6. $7/4$, 7. -2 , 8. 0 ,
9. 0 , 10. 6 , 11. $1/9$, 12. $-7/6$,
13. $23/5$, 14. 6 , 15. -5 , 16. $37/19$,
17. -30 , 18. $3/4$, 19. $-5/3$, 20. -5 ,
21. 7 , 22. $\sqrt{8} - 4$, 23. $\sqrt{23} + 4$, 24. -49 ,
25. $12/19$, 26. 42 , 27. 1 , 28. $8/13$,
29. $-7/3$, 30. $13/3$, 31. 15 , 32. $7/6$,
33. $-2/5$, 34. $x = 9/200$, 35. $s = 13$.

Section 7. Transposition of formulae

1. $t = \frac{c^2}{p^2}$, 2. $N = \sqrt{\frac{L\ell}{\mu A}}$,
3. a) $e = \frac{h-c-d}{2}$, b) $h = \frac{S-2\pi r^2}{2\pi r}$, c) $c = \frac{d(1+Q^2)}{Q^2-1}$ d) $x = \frac{21-5y}{2}$,
4. $n = \frac{mJ}{E-LJ}$

Section 8. Solving quadratic equations by factorisation

1. $1, 2$, 2. $-1, 2$, 3. $-2, 1$, 4. $-1, -2$,
5. $-7, -1$, 6. $4, 3$, 7. $-4, 5$, 8. $1, -1$,
9. 1 twice, 10. -1 twice, 11. $-11, 0$, 12. $0, -1$,
13. $0, 3$, 14. $0, -9$, 15. $2, \frac{1}{2}$, 16. $\frac{1}{2}, -\frac{1}{3}$,
17. $\frac{1}{5}, 1$, 18. $1, 3$.

Section 9. Solving quadratic equations by using a standard formula and by completing the square

Note that answers were requested in surd form. Decimal approximations are not acceptable.

1. $-4 \pm \sqrt{15}$, 2. $-\frac{7}{2} \pm \frac{\sqrt{57}}{2}$, 3. $-3 \pm \sqrt{11}$, 4. $-\frac{3}{8} \pm \frac{\sqrt{41}}{8}$,
5. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$, 6. $-\frac{1}{2} \pm \frac{\sqrt{5}}{2}$, 7. $\frac{3}{2} \pm \frac{\sqrt{13}}{2}$, 8. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$,
9. $\frac{1}{2}, -3$, 10. $-3/2, 1$, 11. $-\frac{8}{9} \pm \frac{\sqrt{55}}{9}$, 12. $-8 \pm \sqrt{55}$.

Section 10. Solving simultaneous linear equations

1. $x = 3$, $y = -2$ 4. $a = 5/2$, $y = -2$ 7. $a = 4$, $b = -1$
2. $x = 1$, $y = 2$ 5. $x = -1$, $y = 3$ 8. $x = -3$, $y = 5$
3. $x = 3$, $y = 1/2$ 6. $x = 3/2$, $y = -2$ 9. $x = 9/2$, $y = -3$

Mock Assessment

The test you will sit in September will ask questions similar to this one. Be sure you are able to answer these questions well! You should also be able to complete this test within 1 hour.

You may NOT use a calculator

Useful Formulae If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Expand and simplify

(a) $(x + 3)(x - 1)$

(b) $(a + 3)^2$

(c) $(2x + 3)(x - 4)$

2. Factorise

(a) $x^2 - 7x$

(b) $y^2 - 64$

(c) $x^2 - 5x + 6$

(d) $6t^2 - 13t + 5$

3. Simplify

(a) $\frac{4x^3y}{8x^2y^3}$

(b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4. Solve the following equations

(a) $\frac{h-1}{4} + \frac{3h}{5} = 4$

(b) $x^2 - 8x = 0$

(c) $p^2 + 4p - 12 = 0$

5. Write each of the following as single powers of x and/or y

(a) $y^3 \times y^2$

(b) $\frac{1}{x^4}$

(c) $(x^2y)^3$

(d) $\frac{x^5}{x^{-2}}$

6. Work out the values of the following, giving your answers as fractions

(a) 4^{-2}

(b) 10^0

(c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations

$$3x - 5y = -11$$

$$5x - 2y = 7$$

8. Rearrange the following equations to make x the subject

(a) $v^2 = u^2 + 2ax$

(b) $V = \frac{1}{3}\pi x^2 h$

(c) $y = \frac{x+2}{x+1}$

9. Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

Mock Assessment – Answers

1) a) $x^2 + 2x - 3$ b) $a^2 + 6a + 9$ c) $2x^2 - 5x - 12$

2) a) $x(x - 7)$ b) $(y + 8)(y - 8)$ c) $(x - 2)(x - 3)$ d) $(3t - 5)(2t - 1)$

3) a) $\frac{x}{2y^2}$ b) $\frac{10x + 3}{6}$

4) a) $h = 5$ b) $x = 0$ or $x = 8$ c) $p = -6$ or $p = 2$

5) a) y^5 b) x^4 c) x^6y^3 d) x^7

6) a) $\frac{1}{16}$ b) 1 c) $\frac{2}{3}$

7) $x = 3, y = 4$

8) a) $x = \frac{v^2 - u^2}{2a}$ b) $x = \sqrt{\frac{3V}{\pi h}}$ c) $x = \frac{2 - y}{y - 1}$

9) $x = \frac{1 \pm \sqrt{21}}{10}$