1. Arithmetic of Fractions

1.   Express each of the following as a fraction in its simplest form. For example $\frac{3}{21}$ can be written as $\frac{1}{7}$. Remember, no calculators!

a) $\frac{20}{45}$ b) $\frac{16}{36}$ c) $-\frac{42}{21}$ d) $\frac{18}{16}$ e) $\frac{30}{30}$ f) $\frac{17}{21}$ g) $-\frac{49}{35}$ h) $\frac{90}{30}$

2.   Calculate

a) $\frac{1}{2}+\frac{1}{3}$ b) $\frac{1}{2}-\frac{1}{3}$ c) $\frac{2}{3}+\frac{3}{4}$ d) $\frac{5}{6}-\frac{2}{3}$ e) $\frac{8}{9}+\frac{1}{5}+\frac{1}{6}$ f) $\frac{4}{5}+\frac{3}{7}-\frac{9}{10}$

3.   Evaluate the following, expressing each answer in its simplest form.

a) $\frac{4}{5}\times\frac{3}{16}$ b) $2\times3\times\frac{1}{4}$ c) $\frac{3}{4}\times\frac{3}{4}$ d) $\frac{4}{9}\times6$ e) $\frac{15}{16}\times\frac{4}{5}$ f) $\frac{9}{5}\times\frac{1}{3}\times\frac{15}{27}$

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4. Evaluate

a) $3 \div \frac{1}{2}$ b) $\frac{1}{2} \div \frac{1}{4}$ c) $\frac{6}{7} \div \frac{16}{21}$ d) $\frac{\frac{3}{4}}{4}$ e) $5 \div \frac{10}{9}$ f) $\frac{3}{4} \div \frac{4}{3}$

5. Express the following as mixed fractions. A mixed fraction has a whole number part and a fractional part. For example, $\frac{13}{5}$ can be written as the mixed fraction $2\frac{3}{5}$.

a) $\frac{5}{2}$ b) $\frac{7}{3}$ c) $-\frac{11}{4}$ d) $\frac{6}{5}$ e) $\frac{12}{5}$ f) $\frac{18}{7}$ g) $\frac{16}{3}$ h) $\frac{83}{9}$

6. Express the following as improper fractions. An improper fraction is 'top-heavy'. Its numerator is greater than its denominator. For example, the mixed fraction $13\frac{4}{5}$ can be written as the improper fraction $\frac{69}{5}$.

a) $2\frac{1}{4}$ b) $3\frac{1}{2}$ c) $5\frac{2}{3}$ d) $-3\frac{2}{5}$ e) $11\frac{4}{6}$ f) $8\frac{2}{9}$ g) $16\frac{3}{4}$ h) $89\frac{2}{7}$

2. Rules and Manipulation of Indices

1. Simplify the following algebraic expressions.

   a) $x^3 \times x^4$ b) $y^2 \times y^3 \times y^5$ c) $z^3 \times z^2 \times z$ d) $t^2 \times t^{10} \times t$

   e) $a \times a \times a^2$ f) $t^3 t^4$ g) $b^6 b^3 b$ h) $z^7 z^7$

2. Simplify

   a) $\dfrac{x^6}{x^2}$ b) $\dfrac{y^{14}}{y^{10}}$ c) $\dfrac{t^{16}}{t^{12}}$ d) $\dfrac{z^{10}}{z^9}$ e) $\dfrac{v^7}{v^0}$ f) x^7/x^4

3. Simplify the following:

   a) $\dfrac{10^7}{10^6}$ b) $\dfrac{10^{19}}{10^{16}}$ c) $\dfrac{x^7}{x^{14}}$ d) $\dfrac{x^7}{y^4}$

   e) $\dfrac{(ab)^4}{a^2b^2}$ f) $\dfrac{9^9 10^{10}}{10^9}$ g) $\dfrac{x^9 y^8}{y^7 x^6}$,   h) $\dfrac{(abc)^3}{(abc)^2}$

4. Write the following expressions using only *positive* indices. For example $\dfrac{x^{-4}}{x^{-2}}$ can be written as $\dfrac{1}{x^2}$.

a) $x^{-2}x^{-1}$

b) $\dfrac{3x}{x^{-4}}$

c) $\dfrac{t^{-2}}{t^{-3}}$

d) $(2a^2b^3)(6a^{-3}b^{-5})$

e) $\dfrac{x^{-3}}{5^{-2}}$

f) $\dfrac{(27)^{-1}x^{-1}}{y^{-2}}$

5. Without using a calculator, evaluate

a) $\dfrac{3}{4^{-2}}$

b) 4×3^{-2}

c) $3^{-1}9^2(27)^{-1}$

d) $(0.25)^{-1}$

e) $(0.2)^{-2}$

f) $(0.1)^{-3}$

6. Simplify

a) $t^{-6}t^3$

b) $(-y^{-2})(-y^{-1})$

c) $\dfrac{3y^{-2}}{6y^{-3}}$

d) $(-2t^{-1})(-3t^{-2})(-4t^{-3})$

e) $\dfrac{3t^{-2}}{6t^3}$

f) $\dfrac{(2t^{-1})^3}{6t^2}$

g) $\dfrac{(-2t)^3}{(-4t)^2}$

7. Write the following expressions using a *single* index. For example $(5^3)^{-4}$ can be written as 5^{-12}.

a) $(5^3)^5$ b) $(3^3)^3$ c) $(17^2)^4$ d) $(y^3)^6$ e) $(\frac{y^{-1}}{y^{-2}})^3$

f) $(\frac{t^{-2}}{t^4})^3$ g) $(k^{-2})^{-6}$ h) $((-1)^4)^3$ i) $((-1)^{-4})^{-3}$

8. Without the use of a calculator, evaluate

a) $(4^{-1})^2$ b) $(2^2)^{-1}$ c) $(3^2)^2$ d) $(6^{-2})^{-1}$

e) $\left(\frac{2}{5^2}\right)^{-1}$ f) $(-2)^{-1}$ g) $\left(-\frac{2}{3}\right)^{-2}$

9. Write the following expressions without using brackets.

a) $(4^2 5^3)^3$

b) $\left(\dfrac{3ab}{c^3}\right)^2$

c) $\left(\dfrac{4^{-2}a^{-3}}{b^{-1}}\right)^2$

d) $(2a^2 b)^3$

e) $(3xy^2 z^3)^2$

f) $\left(\dfrac{6}{ab^2}\right)^2$

g) $\left(-\dfrac{3}{x^2}\right)^2$

h) $\left(\dfrac{2z^2}{3t}\right)^3$

i) $(-2x)^2$

j) $(-2x^2)^{-2}$

k) $\left(-\dfrac{2}{x^2}\right)^{-3}$

10. Write the following expressions without using brackets.

a) $(6^{1/2})^3$

b) $(5^{1/3})^6$

c) $(10^{0.6})^4$

d) $(x^2)^{1/3}$

e) $(2x^2)^{1/3}$

f) $(a \times a^2)^{1/2}$

g) $(ab^2)^{1/2}$

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11. Write the following expressions without using brackets.

a) $(4^3)^{-1/2}$ b) $(3^{-1/2})^{-1/2}$ c) $(7^{2/3})^4$ d) $(19^{3/2})^{1/3}$

e) $(a^2 b^{-3})^{-\frac{3}{2}}$ f) $\left(\dfrac{k^{-1.5}}{\sqrt{k}}\right)^{-2}$

12. Write the following expressions without using brackets.

a) $(5b)^{1/6}$ b) $(3\sqrt{x})^3$ c) $3(\sqrt{x})^3$ d) $(\sqrt{3x})^3$

13. Simplify

a) $x^{1/2} x^{1/3}$ b) $\dfrac{x^{1/2}}{x^{1/3}}$ c) $(x^{1/2})^{1/3}$ d) $(8x^3)^{1/3}$

e) $\sqrt{25y^2}$ f) $\left(\dfrac{27}{t^3}\right)^{1/3}$ g) $(16y^4)^{1/4}$ h) $\left(x^{1/4} x^{1/2}\right)^4$

i) $\sqrt{a^2 a^6}$ j) $\sqrt{\dfrac{a^{-4}}{a^{-1}}}$

3. Expanding Brackets and Factorising

MyMaths.co.uk **Algebra > Algebraic Manipulation > Single Brackets, Brackets, Factorising Linear, Factorising Quadratics 1 and 2**

1. Write the following expressions without using brackets:

a) $2(mn)$ b) $2(m+n)$ c) $a(mn)$ d) $a(m+n)$ e) $a(m-n)$

f) $(am)n$ g) $(a+m)n$ h) $(a-m)n$ i) $5(pq)$ j) $5(p+q)$

k) $5(p-q)$ l) $7(xy)$ m) $7(x+y)$ n) $7(x-y)$ o) $8(2p+q)$

p) $8(2pq)$ q) $8(2p-q)$ r) $5(p-3q)$ s) $5(p+3q)$ t) $5(3pq)$

2. Write the following expressions without using brackets and simplify where possible:

a) $(2+a)(3+b)$ b) $(x+1)(x+2)$ c) $(x+3)(x+3)$ d) $(x+5)(x-3)$

3. Write the following expressions without using brackets:

a) $(7+x)(2+x)$ b) $(9+x)(2+x)$ c) $(x+9)(x-2)$ d) $(x+11)(x-7)$

e) $(x+2)x$ f) $(3x+1)x$ g) $(3x+1)(x+1)$ h) $(3x+1)(2x+1)$

i) $(3x+5)(2x+7)$ j) $(3x+5)(2x-1)$ k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$

4. Rewrite the following expressions without using brackets:

a) $(s+1)(s+5)(s-3)$ b) $(x+y)^3$

5. Factorise

a) $5x+15y$ b) $3x-9y$ c) $2x+12y$ d) $4x+32z+16y$ e) $\frac{1}{2}x+\frac{1}{4}y$

6. Factorise

a) $\frac{1}{3}x + \frac{1}{6}xy$ b) $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ c) $a^2 - a + \frac{1}{4}$ d) $\frac{1}{x^2} - \frac{2}{x} + 1$

7. Factorise

a) $x^2 + 8x + 7$ b) $x^2 + 6x - 7$ c) $x^2 + 7x + 10$ d) $x^2 - 6x + 9$ e) $x^2 + 5x + 6$.

8. Factorise

a) $2x^2 + 3x + 1$ b) $2x^2 + 4x + 2$ c) $3x^2 - 3x - 6$ d) $5x^2 - 4x - 1$

e) $16x^2 - 1$ f) $-x^2 + 1$ g) $-2x^2 + x + 3$

9. Factorise

a) $x^2 + 9x + 14$ b) $x^2 + 11x + 18$ c) $x^2 + 7x - 18$ d) $x^2 + 4x - 77$

e) $x^2 + 2x$ f) $3x^2 + x$,   g) $3x^2 + 4x + 1$ h) $6x^2 + 5x + 1$

i) $6x^2 + 31x + 35$ j) $6x^2 + 7x - 5$ k) $-3x^2 + 2x + 5$ l) $x^2 - 3x + 2$

10. Rewrite the following expressions without using brackets, simplifying where possible:

a) $15 - (7 - x)$ b) $15 - 7(1 - x)$

c) $15 - 7(x - 1)$ d) $(2x - y) - x(1 + y)$

e) $x(a + b) - x(a + 3b)$ f) $2(5a + 3b) + 3(a - 2b)$

g) $-(4a + 5b - 3c) - 2(2a + 3b - 4c)$ h) $2x(x - 5) - x(x - 2) - 3x(x - 5)$

11. Rewrite each of the following expressions without using brackets and simplify where possible

a) $2x-(3y+8x)$,   b) $2x+5(x-y-z)$,   c) $-(5x-3y)$,   d) $5(2x-y)-3(x+2y)$

4. Algebraic Fractions

1. Express each of the following as a single fraction.

a) $2 \times \dfrac{x+y}{3}$ b) $\dfrac{1}{3} \times 2(x+y)$ c) $\dfrac{2}{3} \times (x+y)$

2. Simplify

a) $3 \times \dfrac{x+4}{7}$ b) $\dfrac{1}{7} \times 3(x+4)$ c) $\dfrac{3}{7} \times (x+4)$ d) $\dfrac{x}{y} \times \dfrac{x+1}{y+1}$

e) $\dfrac{1}{y} \times \dfrac{x^2+x}{y+1}$ f) $\dfrac{\pi d^2}{4} \times \dfrac{Q}{\pi d^2}$ g) $\dfrac{Q}{\pi d^2/4}$ h) $\dfrac{1}{x/y}$

3. Simplify   a) $\dfrac{6/7}{s+3}$ b) $\dfrac{3/4}{x-1}$ c) $\dfrac{x-1}{3/4}$

4. Simplify $\dfrac{3}{x+2} \div \dfrac{x}{2x+4}$

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5. Simplify $\dfrac{5}{2x+1} \div \dfrac{x}{3x-1}$

6. Simplify

a) $\dfrac{x}{4} + \dfrac{x}{7}$

b) $\dfrac{2x}{5} + \dfrac{x}{9}$

c) $\dfrac{2x}{3} - \dfrac{3x}{4}$

d) $\dfrac{x}{x+1} - \dfrac{2}{x+2}$

e) $\dfrac{x+1}{x} + \dfrac{3}{x+2}$

f) $\dfrac{2x+1}{3} - \dfrac{x}{2}$

g) $\dfrac{x+3}{2x+1} - \dfrac{x}{3}$

h) $\dfrac{x}{4} - \dfrac{x}{5}$

7. Simplify

a) $\dfrac{1}{x+2} + \dfrac{2}{x+3}$

b) $\dfrac{2}{x+3} + \dfrac{5}{x+1}$

c) $\dfrac{2}{2x+1} - \dfrac{3}{3x+2}$

d) $\dfrac{x+1}{x+3} + \dfrac{x+4}{x+2}$

e) $\dfrac{x-1}{x-3} + \dfrac{x-1}{(x-3)^2}$

5. Surds

Roots, for example $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{6}$ are also known as surds. A common cause of error is misuse of expressions involving surds. You should be aware that $\sqrt{ab} = \sqrt{a}\sqrt{b}$ but $\sqrt{a+b}$ is NOT equal to $\sqrt{a} + \sqrt{b}$.

1.   It is often possible to write surds in equivalent forms. For example $\sqrt{48}$ can be written $\sqrt{3 \times 16} = \sqrt{3} \times \sqrt{16} = 4\sqrt{3}$.

Write the following in their simplest forms:

a) $\sqrt{180}$ b) $\sqrt{63}$

2.   By multiplying numerator and denominator by $\sqrt{2}+1$ show that

$$\frac{1}{\sqrt{2}-1} \qquad \text{is equivalent to} \qquad \sqrt{2}+1$$

3.   Simplify, if possible,   a) $\sqrt{x^2 y^2}$ b) $\sqrt{x^2 + y^2}$.

4. Study the following expressions and simplify where possible.

a) $\sqrt{(x+y)^4}$ b) $(\sqrt[3]{x+y})^6$ c) $\sqrt{x^4+y^4}$

5. By considering the expression $(\sqrt{x}+\sqrt{y})^2$ show that

$$\sqrt{x}+\sqrt{y} = \sqrt{x+y+2\sqrt{xy}}$$

Find a corresponding expression for $\sqrt{x}-\sqrt{y}$.

6. Write each of the following as an expression under a single square root sign. (For parts c) and d) see Question 5 above.)

a) $2\sqrt{p}$ b) $\sqrt{p}\sqrt{q^3}$ c) $\sqrt{p}+\sqrt{2q}$ d) $\sqrt{3}-\sqrt{2}$

7. Use indices (powers) to write the following expressions without the root sign.

a) $\sqrt[4]{a^2}$ b) $(\sqrt{3}\times\sqrt{5})^3$

6. Linear Equations

In questions $1 - 35$ solve each equation:

1. $3y - 8 = \frac{1}{2}y$ 2. $7t - 5 = 4t + 7$ 3. $3x + 4 = 4x + 3$ 4. $4 - 3x = 4x + 3$

5. $3x + 7 = 7x + 2$ 6. $3(x + 7) = 7(x + 2)$ 7. $2x - 1 = x - 3$ 8. $2(x + 4) = 8$

9. $-2(x - 3) = 6$ 10. $-2(x - 3) = -6$

11. $-3(3x - 1) = 2$ 12. $2 - (2t + 1) = 4(t + 2)$

13. $5(m - 3) = 8$ 14. $5m - 3 = 5(m - 3) + 2m$

15. $2(y + 1) = -8$ 16. $17(x - 2) + 3(x - 1) = x$

17. $\dfrac{1}{3}(x+3) = -9$ 18. $\dfrac{3}{m} = 4$ 19. $\dfrac{5}{m} = \dfrac{2}{m+1}$ 20. $-3x + 3 = 18$

21. $3x + 10 = 31$ 22. $x + 4 = \sqrt{8}$ 23. $x - 4 = \sqrt{23}$

24. $\dfrac{x-5}{2} - \dfrac{2x-1}{3} = 6$ 25. $\dfrac{x}{4} + \dfrac{3x}{2} - \dfrac{x}{6} = 1$

26. $\dfrac{x}{2} + \dfrac{4x}{3} = 2x - 7$ 27. $\dfrac{5}{3m+2} = \dfrac{2}{m+1}$

28. $\dfrac{2}{3x-2}=\dfrac{5}{x-1}$ **29.** $\dfrac{x-3}{x+1}=4$ **30.** $\dfrac{x+1}{x-3}=4$ **31.** $\dfrac{y-3}{y+3}=\dfrac{2}{3}$

32. $\dfrac{4x+5}{6}-\dfrac{2x-1}{3}=x$ **33.** $\dfrac{3}{2s-1}+\dfrac{1}{s+1}=0$

34. $\dfrac{1}{5x}+\dfrac{1}{4x}=10.$ **35.** $\dfrac{3}{s-1}=\dfrac{2}{s-5}.$

7. Changing the Subject of a Formulae

1. Make t the subject of the formula $p = \dfrac{c}{\sqrt{t}}$.

2. Make N the subject of the formula $L = \dfrac{\mu N^2 A}{\ell}$.

3. In each case make the specified variable the subject of the formula:

   a) $h = c + d + 2e$, e b) $S = 2\pi r^2 + 2\pi rh$, h

   c) $Q = \sqrt{\dfrac{c+d}{c-d}}$, c d) $\dfrac{x+y}{3} = \dfrac{x-y}{7} + 2$, x

4. Make n the subject of the formula $J = \dfrac{nE}{nL+m}$.

8. Solving Quadratic Equations – Factorising

Solve the following equations by factorisation:

1. $x^2 - 3x + 2 = 0$ 2. $x^2 - x - 2 = 0$ 3. $x^2 + x - 2 = 0$ 4. $x^2 + 3x + 2 = 0$

5. $x^2 + 8x + 7 = 0$ 6. $x^2 - 7x + 12 = 0$ 7. $x^2 - x - 20 = 0$ 8. $x^2 - 1 = 0$

9. $x^2 - 2x + 1 = 0$ 10. $x^2 + 2x + 1 = 0$ 11. $x^2 + 11x = 0$ 12. $2x^2 + 2x = 0$

13. $x^2 - 3x = 0$ 14. $x^2 + 9x = 0$ 15. $2x^2 - 5x + 2 = 0$ 16. $6x^2 - x - 1 = 0$

17. $-5x^2 + 6x - 1 = 0$ 18. $-x^2 + 4x - 3 = 0$

9. Solving Quadratic Equations – Completing the Square and Using the Quadratic Formula

Solve each of the following quadratic equations twice: once by using the formula, then again by completing the square. Obtain your answers in surd, not decimal, form.

1. $x^2 + 8x + 1 = 0$ 2. $x^2 + 7x - 2 = 0$ 3. $x^2 + 6x - 2 = 0$

4. $4x^2 + 3x - 2 = 0$ 5. $2x^2 + 3x - 1 = 0$ 6. $x^2 + x - 1 = 0$

7. $-x^2 + 3x + 1 = 0$ 8. $-2x^2 - 3x + 1 = 0$ 9. $2x^2 + 5x - 3 = 0$

10. $-2s^2 - s + 3 = 0$ 11. $9x^2 + 16x + 1 = 0$ 12. $x^2 + 16x + 9 = 0$

13. Show that the roots of $x^2 - 2x + \alpha = 0$ are $x = 1 + \sqrt{1 - \alpha}$ and $x = 1 - \sqrt{1 - \alpha}$.

14. Show that the roots of $x^2 - 2\alpha x + \beta = 0$ are

$$x = \alpha + \sqrt{\alpha^2 - \beta} \quad \text{and} \quad x = \alpha - \sqrt{\alpha^2 - \beta}$$

10. Solving Simultaneous Linear Equations

Solve each of the pairs of simultaneous equations. Where the answer is not a whole number, give your answer as an improper fraction.

1. $4x + 3y = 6$
    $5x - 3y = 21$

2. $3x + 5y = 13$
    $2x + 3y = 8$

3. $x + 4y = 5$
    $4x - 2y = 11$

4. $2a + b = 3$
    $4a - 5b = 20$

5. $5x + 3y = 4$
    $3x + 4y = 9$

6. $6x - 2y = 13$
    $2x + 3y = -3$

7. $3a - 2b = 14$
    $4a + 3b = 13$

8. $5x + 4y = 5$
    $2x + 7y = 29$

9. $6x - 4y = 39$
    $2x + y = 6$

Answers

Section 1. Arithmetic of fractions

1. a) $\frac{4}{9}$, b) $\frac{4}{9}$, c) -2, d) $\frac{9}{8}$, e) 1, f) $\frac{17}{21}$, g) $-\frac{7}{5}$, h) 3
2. a) $\frac{5}{6}$, b) $\frac{1}{6}$, c) $\frac{17}{12}$, d) $\frac{1}{6}$, e) $\frac{113}{90}$, f) $\frac{23}{70}$
3. a) $\frac{3}{20}$, b) $\frac{3}{2}$, c) $\frac{9}{16}$, d) $\frac{8}{3}$, e) $\frac{3}{4}$, f) $\frac{1}{3}$.
4. a) 6, b) 2, c) $\frac{9}{8}$, d) $\frac{3}{16}$, e) $\frac{9}{2}$, f) $\frac{9}{16}$
5. a) $2\frac{1}{2}$, b) $2\frac{1}{3}$, c) $-2\frac{3}{4}$, d) $1\frac{1}{5}$, e) $2\frac{2}{5}$, f) $2\frac{4}{7}$, g) $5\frac{1}{3}$, h) $9\frac{2}{9}$
6. a) $\frac{9}{4}$, b) $\frac{7}{2}$, c) $\frac{17}{3}$, d) $-\frac{17}{5}$, e) $\frac{35}{3}$, f) $\frac{74}{9}$, g) $\frac{67}{4}$, h) $\frac{625}{7}$

Section 2. Manipulation of expressions involving indices

1. a) x^7, b) y^{10}, c) z^6, d) t^{13}, e) a^4, f) t^7, g) b^{10}, h) z^{14}.
2. a) x^4, b) y^4, c) t^4, d) z, e) v^7, f) x^3
3. a) 10, b) 10^3, c) x^{-7}, d) $\frac{x^7}{y^4}$, e) $a^2 b^2$, f) $9^9 \cdot 10$, g) $x^3 y$, h) abc
4. a) $\frac{1}{x^3}$, b) $3x^5$, c) t, d) $\frac{12}{ab^2}$, e) $\frac{5^2}{x^3}$, f) $\frac{y^2}{27x}$
5. a) 48, b) $\frac{4}{9}$, c) 1, d) 4, e) 25, f) 1000
6. a) t^{-3}, b) y^{-3}, c) $\frac{1}{2}y$, d) $-24t^{-6}$, e) $\frac{1}{2t^5}$, f) $\frac{4t^{-5}}{3}$, g) $-\frac{t}{2}$.
7. a) 5^{15}, b) 3^9, c) 17^8, d) y^{18}, e) y^3, f) t^{-18}, g) k^{12}, h) $(-1)^{12} = 1$, i) $(-1)^{12} = 1$.
8. a) $\frac{1}{16}$, b) $\frac{1}{4}$, c) 81, d) 36, e) $\frac{25}{2}$, f) $-\frac{1}{2}$, g) $\frac{9}{4}$
9. a) $4^6 5^9$, b) $\frac{9a^2 b^2}{c^6}$, c) $\frac{4^{-4}a^{-6}}{b^{-2}} = \frac{b^2}{4^4 a^6}$ d) $8a^6 b^3$, e) $9x^2 y^4 z^6$, f) $\frac{36}{a^2 b^4}$,

    g) $\frac{9}{x^4}$, h) $\frac{8z^6}{27t^3}$, (i) $4x^2$, j) $\frac{1}{4x^4}$, k) $-\frac{x^6}{8}$.
10. a) $6^{3/2}$, b) 5^2, c) $10^{2.4}$, d) $x^{2/3}$, e) $2^{1/3}x^{2/3}$, f) $a^{3/2}$, g) $a^{1/2}b$.
11. a) $4^{-3/2}$, b) $3^{1/4}$, c) $7^{8/3}$, d) $19^{1/2}$, e) $a^{-3}b^{9/2}$, f) k^4.
12. a) $5^{1/6}b^{1/6}$, b) $27x^{3/2}$, c) $3x^{3/2}$, d) $3^{3/2}x^{3/2}$
13. a) $x^{5/6}$, b) $x^{1/6}$, c) $x^{1/6}$, d) $2x$, e) $5y$, f) $\frac{3}{t}$, g) $2y$, h) x^3, i) a^4, j) $a^{-3/2}$

Section 3. Removing brackets and factorisation

1. a) $2mn$, b) $2m + 2n$, c) amn, d) $am + an$, e) $am - an$, f) amn, g) $an + mn$,

    h) $an - mn$, i) $5pq$, j) $5p + 5q$, k) $5p - 5q$, l) $7xy$, m) $7x + 7y$, n) $7x - 7y$,

    o) $16p + 8q$, p) $16pq$, q) $16p - 8q$, r) $5p - 15q$, s) $5p + 15q$, t) $15pq$
2. a) $6 + 3a + 2b + ab$, b) $x^2 + 3x + 2$, c) $x^2 + 6x + 9$, d) $x^2 + 2x - 15$
3. a) $14 + 9x + x^2$, b) $18 + 11x + x^2$, c) $x^2 + 7x - 18$,

    d) $x^2 + 4x - 77$, e) $x^2 + 2x$, f) $3x^2 + x$, g) $3x^2 + 4x + 1$

    h) $6x^2 + 5x + 1$, i) $6x^2 + 31x + 35$, j) $6x^2 + 7x - 5$

    k) $-3x^2 + 2x + 5$,   l) $x^2 - 3x + 2$
4. a) $s^3 + 3s^2 - 13s - 15$, b) $x^3 + 3x^2 y + 3xy^2 + y^3$

5. a) $5(x+3y)$, b) $3(x-3y)$, c) $2(x+6y)$, d) $4(x+8z+4y)$, e) $\frac{1}{2}(x+\frac{1}{2}y)$

6. a) $\frac{x}{3}\left(1+\frac{y}{2}\right)$, b) $\frac{\pi r^2}{3}(2r+h)$, c) $\left(a-\frac{1}{2}\right)^2$, d) $\left(\frac{1}{x}-1\right)^2$.

7. a) $(x+7)(x+1)$, b) $(x+7)(x-1)$, c) $(x+2)(x+5)$, d) $(x-3)(x-3)=(x-3)^2$,
e) $(x+3)(x+2)$

8. a) $(2x+1)(x+1)$, b) $2(x+1)^2$, c) $3(x+1)(x-2)$, d) $(5x+1)(x-1)$,
e) $(4x+1)(4x-1)$, f) $(x+1)(1-x)$, g) $(x+1)(3-2x)$

9. a) $(7+x)(2+x)$,   b) $(9+x)(2+x)$,   c) $(x+9)(x-2)$,   d) $(x+11)(x-7)$,
e) $(x+2)x$,   f) $(3x+1)x$,   g) $(3x+1)(x+1)$,   h) $(3x+1)(2x+1)$ i) $(3x+5)(2x+7)$,
j) $(3x+5)(2x-1)$,   k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$

10. a) $8+x$, b) $8+7x$, c) $22-7x$, d) $x-y-xy$, e) $-2bx$, f) $13a$, g) $-8a-11b+11c$,
h) $7x-2x^2$.

11.   a) $-3y-6x$, b) $7x-5y-5z$, c) $-5x+3y$, d) $7x-11y$.

Section 4. Arithmetic of Algebraic Fractions

1. a) $\frac{2(x+y)}{3}$, b) $\frac{2(x+y)}{3}$, c) $\frac{2(x+y)}{3}$

2.   a) $\frac{3(x+4)}{7}$, b) $\frac{3(x+4)}{7}$, c) $\frac{3(x+4)}{7}$, d) $\frac{x(x+1)}{y(y+1)}$, e) $\frac{x(x+1)}{y(y+1)}$, f) $Q/4$, g) $\frac{4Q}{\pi d^2}$, h) $\frac{y}{x}$.

3. a) $\frac{6}{7(s+3)}$, b) $\frac{3}{4(x-1)}$, c) $\frac{4(x-1)}{3}$.

4. $\frac{6}{x}$.   5. $\frac{5(3x-1)}{x(2x+1)}$

6. a) $\frac{11x}{28}$, b) $\frac{23x}{45}$, c) $-\frac{x}{12}$, d) $\frac{x^2-2}{(x+1)(x+2)}$, e) $\frac{x^2+6x+2}{x(x+2)}$, f) $\frac{x+2}{6}$, g) $\frac{9+2x-2x^2}{3(2x+1)}$, h) $\frac{x}{20}$.

7. a) $\frac{3x+7}{(x+2)(x+3)}$, b) $\frac{7x+17}{(x+3)(x+1)}$, c) $\frac{1}{(2x+1)(3x+2)}$, d) $\frac{2x^2+10x+14}{(x+3)(x+2)}$, e) $\frac{x^2-3x+2}{(x-3)^2}$

Section 5. Surds

1. a) $\sqrt{180}=\sqrt{36\times 5}=6\sqrt{5}$. b) $\sqrt{63}=\sqrt{9\times 7}=3\sqrt{7}$.

3.   a) xy.   b) Note that $\sqrt{x^2+y^2}$ is NOT equal to $x+y$.

4. a) $(x+y)^2$,   b) $(x+y)^2$,   c) $\sqrt{x^4+y^4}$ is NOT equal to x^2+y^2.

5. $\sqrt{x}-\sqrt{y}=\sqrt{x+y-2\sqrt{xy}}$.

6. a) $\sqrt{4p}$, b) $\sqrt{pq^3}$, c) $\sqrt{p+2q+2\sqrt{2pq}}$, d) $\sqrt{5-2\sqrt{6}}$.

7. a) $a^{1/2}$,   b) $15^{3/2}$

Section 6. Solving linear equations

1. 16/5, 2. 4, 3. 1, 4. 1/7,
5. 5/4, 6. 7/4, 7. −2, 8. 0,
9. 0, 10. 6, 11. 1/9, 12. −7/6,
13. 23/5, 14. 6, 15. −5, 16. 37/19,
17. −30, 18. 3/4, 19. −5/3, 20. −5,
21. 7, 22. $\sqrt{8} - 4$, 23. $\sqrt{23} + 4$, 24. −49,
25. 12/19, 26. 42, 27. 1, 28. 8/13,
29. −7/3, 30. 13/3, 31. 15, 32. 7/6,
33. −2/5, 34. $x = 9/200$. 35. $s = 13$.

Section 7. Transposition of formulae

1. $t = \frac{c^2}{p^2}$, 2. $N = \sqrt{\frac{Lt}{\mu A}}$,

3. a) $e = \frac{h-c-d}{2}$, b) $h = \frac{S-2\pi r^2}{2\pi r}$, c) $c = \frac{d(1+Q^2)}{Q^2-1}$ d) $x = \frac{21-5y}{2}$.

4. $n = \frac{mJ}{E-LJ}$

Section 8. Solving quadratic equations by factorisation

1. 1,2, 2. −1,2, 3. −2,1, 4. −1,−2,
5. −7,−1, 6. 4,3, 7. −4,5, 8. 1,−1,
9. 1 twice, 10. −1 twice 11. −11,0, 12. 0,−1,
13. 0,3, 14. 0,−9, 15. 2,$\frac{1}{2}$, 16. $\frac{1}{2}$, $-\frac{1}{3}$,
17. $\frac{1}{5}$,1. 18. 1,3.

Section 9. Solving quadratic equations by using a standard formula and by completing the square

Note that answers were requested in surd form. Decimal approximations are not acceptable.

1. $-4 \pm \sqrt{15}$. 2. $-\frac{7}{2} \pm \frac{\sqrt{57}}{2}$. 3. $-3 \pm \sqrt{11}$. 4. $-\frac{3}{8} \pm \frac{\sqrt{41}}{8}$.
5. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$. 6. $-\frac{1}{2} \pm \frac{\sqrt{5}}{2}$. 7. $\frac{3}{2} \pm \frac{\sqrt{13}}{2}$. 8. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$.
9. $\frac{1}{2}$,−3. 10. −3/2,1. 11. $-\frac{8}{9} \pm \frac{\sqrt{55}}{9}$. 12. $-8 \pm \sqrt{55}$.

Section 10. Solving simultaneous linear equations

1. $x = 3$, $y = -2$ 4. $a = 5/2$, $y = -2$ 7. $a = 4$, $b = -1$
2. $x = 1$, $y = 2$ 5. $x = -1$, $y = 3$ 8. $x = -3$, $y = 5$
3. $x = 3$, $y = \frac{1}{2}$ 6. $x = 3/2$, $y = -2$ 9. $x = 9/2$, $y = -3$

Mock Assessment

The test you will sit in September will ask questions similar to this one. Be sure you are able to answer these questions well! You should also be able to complete this test within 1 hour.

You may NOT use a calculator

Useful Formulae   If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1.   Expand and simplify

     (a) $(x + 3)(x - 1)$ (b) $(a + 3)^2$ (c) $(2x + 3)(x - 4)$

2.   Factorise

     (a) $x^2 - 7x$ (b) $y^2 - 64$ (c) $x^2 - 5x + 6$ (d) $6t^2 - 13t + 5$

3.   Simplify   (a) $\dfrac{4x^3 y}{8x^2 y^3}$ (b) $\dfrac{3x + 2}{3} + \dfrac{4x - 1}{6}$

4.   Solve the following equations

     (a) $\dfrac{h-1}{4} + \dfrac{3h}{5} = 4$ (b) $x^2 - 8x = 0$ (c) $p^2 + 4p - 12 = 0$

5.   Write each of the following as single powers of x and/or y

     (a) $y^3 \times y^2$ (b) $\dfrac{1}{x^4}$ (c) $(x^2 y)^3$ (d) $\dfrac{x^5}{x^{-2}}$

6.   Work out the values of the following, giving your answers as fractions

     (a) 4^{-2} (b) 10^0 (c) $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$

7.   Solve the simultaneous equations $3x - 5y = -11$
                                             $5x - 2y = 7$

8.   Rearrange the following equations to make x the subject

     (a) $v^2 = u^2 + 2ax$ (b) $V = \dfrac{1}{3}\pi x^2 h$ (c) $y = \dfrac{x+2}{x+1}$

9.   Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

Mock Assessment – Answers

1) a) $x^2 + 2x - 3$ b) $a^2 + 6a + 9$ c) $2x^2 - 5x - 12$

2) a) $x(x-7)$ b) $(y+8)(y-8)$ c) $(x-2)(x-3)$ d) $(3t-5)(2t-1)$

3) a) $\dfrac{x}{2y^2}$ b) $\dfrac{10x+3}{6}$

4) a) $h = 5$ b) $x = 0$ or $x = 8$ c) $p = -6$ or $p = 2$

5) a) y^5 b) x^{-4} c) $x^6 y^3$ d) x^7

6) a) $\dfrac{1}{16}$ b) 1 c) $\dfrac{2}{3}$

7) $x = 3, y = 4$

8) a) $x = \dfrac{v^2 - u^2}{2a}$ b) $x = \sqrt{\dfrac{3V}{\pi h}}$ c) $x = \dfrac{2-y}{y-1}$

9) $x = \dfrac{1 \pm \sqrt{21}}{10}$